

2025학년도 편입학전형

필기고사 문제지

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(영어 / 수학)
(각 20문항 4지선다형)

시험시간 : 90분

지원학부(과) :

수험번호 :

성명 :

한 국 항 공 대 학 교

필기고사 (영어 / 수학)

■ 유의사항

1. 시험시간은 90분입니다.
2. 답안 작성은 반드시 “컴퓨터용 싸인펜”을 사용하십시오.
3. 수정 펜/수정 테이프 사용을 불허하며, 수정이 필요하면 답안지를 교체하십시오.
4. 답안지에 불필요한 사항을 기재(표시)하지 마시오.
5. [] 안의 숫자는 배점입니다.

[1-5] Choose the word that is closest in meaning to the underlined expression. [각 4점]

1. This innocuous remarks were misinterpreted by some of the audience.
 ① inoffensive ② sarcastic ③ insincere ④ banal
2. The argument made by the author was based on a quixotic interpretation of the historical events.
 ① former ② imbalanced ③ impractical ④ flimsy
3. The government's moribund economic reform policy is being reviewed by experts to assess its future potential.
 ① progressive ② thriving ③ controversial ④ stagnant
4. The decision to postpone the meeting was seen as a prudent move, considering the overall situation of the company.
 ① daring ② autocratic ③ deliberate ④ abrupt
5. This trailer does a decent job of obfuscating the major plot points of the film.
 ① concealing ② highlighting ③ simplifying ④ delivering

[6-8] Choose the one that best completes the sentence. [각 4.6점]

6. Beaked hazelnuts are a little sweeter and more buttery than _____ hazelnuts. Nestled in a fuzzy, green husks that extends outward like the beak of a bird, beaked hazelnuts carpet the forest valleys of British Columbia. For generations, First Nations tribes in the region have passed down stories of these hazelnuts as a vital food source their ancestors planted and cultivated. These stories motivated researchers to look more deeply at hazelnut genetics, in the hopes of determining just how widely the hazelnut was cultivated. The team visited the archaeological remains of villages throughout British Columbia and sampled over 200 hazelnuts nearby. They determined that beaked hazelnuts had been actively cultivated across a wide span of regional territory, up to 800 kilometers away. This genetic detective work verifies how First Nations people changed the forest in long-lasting ways.

- ① nutritious ② alien ③ provincial ④ exported

7. Before conducting a recent survey of Americans to figure out how much they're using generative AI, David Deming, an economist, says he was skeptical that the explosion of generative AI would offer sizable benefits for the U.S. economy anytime soon. Now, however, he says he's more optimistic. The study was motivated by questions over whether and how much Americans are using generative AI. Deming says he was shocked by the results. They found that almost 40% of Americans, ages 18 to 64, have used generative AI. And a sizable percentage seems to use it regularly. The economists found that more than 24% of American workers had used it at least once in the week prior to being surveyed, and nearly one in nine used it every workday. The usage of AI seems to be pretty much _____. "We even found that 22% of blue-collar workers say they use AI, and usage rates were above 20% in every major occupation category except personal services, where it was like 15%," he says.

- ① exceptional ② one-sided ③ around the corner ④ across the board

8. Pantone's Color of the Year is meant to capture the zeitgeist, said the vice president of the Pantone Color Institute. At the same time, it's also intended to serve as _____. Before it became the color juggernaut it is today, Pantone was a commercial printing company under a different name. When Lawrence Herbert, a print technician with a background in chemistry, was hired by the corporation in the 1950s, he identified a recurring problem in his work. When requesting printed copies, of brochures or posters, customers struggled to talk about color accurately. To get the color they were after, they would have to send in an actual sample of the color. One famous thing was to cut a piece off their tie and send it into the print and say, match this color. They had their own ink formula books, and they could get close. But it was very random. In 1963, Lawrence founded the solution. He developed the Pantone Matching System as a way to standardize color reproduction so that print copies matched the original, no matter the printing device. Pantone expanded its range of pigments, and by 1968 it became the industry standard. People credits the shrewd marketer in Herbert for turning Pantone into a widely accepted color system.

- ① a dismal herald ② a marketing trickery
③ a cultural antidote ④ a technological mistake

[9] Choose the one that is either ungrammatical or unacceptable.

[5점]

9. The Boston Mint had defied the British crown's authority to produce coins, representing New England's growing sense of identity as separate from the mother country and its determination to regulate its own economy, according to the Massachusetts Historical Society. After the American Revolution, the coins it ① had produced became vogue even in England. English collector Thomas

Brand Hollis wrote to then-American ambassador to the Netherlands John Adams in 1781 asking for help in sourcing one of these coins. In turn, Adams wrote to ask his wife, Abigail, for help ② although her great-grandfather had been the stepbrother of John Hull, the silversmith who minted these coins. Just ③ one other threepence coin of this type is known to have survived to the present day and it is in the collection of the Massachusetts Historical Society, making this specimen the only one available to private collectors. Another coin is perhaps still in existence after it was stolen from Yale College sometime before the 1960s, although its whereabouts is unknown. Other historic US coins have fetched vast sums at auction. A rare 1794 silver dollar believed to be one of the first - if not the first - made by the US mint sold for \$10 million in 2013. ④ Meanwhile, a rare 1933 “Double Eagle” coin, one of the last gold coins ever struck for circulation in the US, sold for \$18.9 million in 2021.

[10-13] Choose the best for the blank.

[각 4.8점]

10. Perhaps the greatest mind-bending quirk of our universe is the inherent trouble with timekeeping: Seconds tick by ever so slightly faster atop a mountain than they do in the valleys of Earth. For practical purposes, most people don't have to worry about those differences. But a renewed space race has the United States and its allies, dashing to create permanent settlements on the moon, and that has brought the idiosyncrasies of time, once again, to the forefront. On the lunar surface, a single Earth day would be roughly 56 microseconds shorter than on our home planet - a tiny number that can lead to significant inconsistencies over time. NASA and its international partners are currently grappling with the perplexing conundrum. Scientists aren't just looking to create a new time zone on the moon, as some headlines have suggested. Rather, the space agency and its partners are looking to _____ or system of measurement that accounts for that fact that seconds tick by faster on the moon.

- ① separate time zones in space
- ② create an entirely new time scale
- ③ reevaluate the current timekeeping method
- ④ erase the time tracker

11. An odd little gourd called the squirting cucumber has an explosive reproductive strategy that has intrigued naturalists since the days of the Roman Empire. Squirting cucumbers spur their seeds over distances hundreds of times their length, and now scientists know how the plants do it. A squirting cucumber in action is a remarkable sight, but if you blink, you might miss it - the fountain lasts about 0.03 second. When ripe, the hairy green fruit measures about 4 centimeters long. They are members of the gourd family and relatives of zucchini, squash and pumpkins, but

_____. Recently, researchers blew this ballistic mystery wide open. They used high-speed video and digital 3D reconstructions to analyze squirting cucumbers before, during and after a squirt and created mathematical models to describe the fruits' geyser-like eruptions.

- ① they rely on environmental forces to disperse seeds
- ② they exhibit a characteristic of continuous eruptions
- ③ they have a distinct eruptive reproductive mechanism
- ④ the family members employ a retrogressive dispersal strategy

12. Humans' love affair with carbs may _____, according to a new study. A once prevailing stereotype of ancient humans feasting on mammoth steak and other hunks of meat helped foster the idea of a protein-heavy diet that was necessary to fuel the development of a large brain. But archaeological evidence in recent years has challenged this view, suggesting that humans long ago developed a taste for carbohydrates, roasting things such as tubers and other starch-laden foods that have been detected by analyzing bacteria lodged in teeth. The new research offers the first hereditary evidence for early carb-laden diets. Scientists traced the evolution of a gene that enables humans to digest starch more easily by breaking it down into simple sugars that our bodies can use for energy. The study revealed these genes duplicated long before the advent of agriculture. This expansion may even go back hundreds of thousands of years, long before our species, Homo sapiens, or even Neanderthals emerged as distinct human lineages.

- ① predate our existence as a species
- ② have been driven by environmental factors
- ③ explain the origin of human evolution
- ④ begin with a backlash against protein-rich diets

13. A graduate student at MIT made a startling discovery: The facial recognition software program she was working on couldn't detect her dark skin; it only registered her presence when she put on a white mask. It was her first encounter with what she came to call the coded gaze. "You've likely heard of the male gaze or the white gaze," she explains. This is a cousin concept really, about who has the power to shape technology and whose preferences, priorities and prejudices are baked in. She notes that in a recent test of Stable Diffusion's text-to-image generative AI system, prompts for high paying jobs overwhelmingly yielded images of men with lighter skin. Meanwhile, prompts for criminal stereotypes, such as drug dealers, terrorists or inmates, typically resulted in images of men with darker skin. In her new book, she looks at social implications of technology and warns that biases in facial analysis systems could harm millions of people - especially if they reinforce existing stereotypes. "With the adoption of AI systems, at first I thought we were looking at a mirror, but now I believe we're looking into a kaleidoscope of distortion," she says. "Because

the technologies we believe to be bringing us into the future are actually _____.”

- ① making us overly reliant on automation
- ② leading us astray by fostering shallow connections
- ③ building on the advancements we’ve made so far
- ④ taking us back from the progress already made

[14-20] Read the following passages and answer the questions.

[각 6점]

[14-15]

A new report from the National Academy of Sciences says it’s hard to know how many people in the U.S. actually have food allergies or whether they’re on the rise. Part of the challenge is this: Food allergies are often self-diagnosed and symptoms can be misinterpreted. (A) Sometimes people can’t distinguish a food allergy from other conditions such as lactose intolerance or gluten sensitivity, which don’t fit the medical definition of an allergy. There are a lot of misconceptions about what a food allergy is, says Dr. Virginia Stallings. (B) One scenario is this: A parent of a young child introduces a new food – say, milk – into the diet, and then notices the child has an upset stomach or other symptoms of gastrointestinal distress. The parent may suspect a food allergy. But, perhaps, these are signs of lactose intolerance – a completely different condition. The reason food allergy symptoms are often confused with other conditions such as lactose intolerance is because there’s an overlap in some of the symptoms, Stallings explains. Adding to the confusion, food allergies can be complicated to diagnose. (C) There isn’t one skin test or blood test that can accurately determine whether a person has an allergy to a specific food, says Stallings. She adds if parents recognize clear signs of allergic reactions, such as swelling lips or difficulty breathing, they should seek emergency care. (D) But often times, the symptoms are milder. In these cases, rather than self-diagnosing an allergy, families should see an expert, such as a pediatric allergist.

14. Find the best place for the following sentence.

We do not have an optimal diagnostic tool.

- ① (A)
- ② (B)
- ③ (C)
- ④ (D)

15. What would be the best title of the passage above?

- ① Disentangling the complexity behind food allergies
- ② Simplifying the definition of food allergies
- ③ Authorizing a diagnostic tool for food allergies
- ④ Recognizing symptoms to self-diagnose food allergies

[16]

The roots of America's candy boom lie in the 1920s. Sugar trade routes that had been disrupted during World War I were once again open for business. The result: a glut of sugar that led to a steep crash in prices. This was a mess for farmers in the mainland United States, Cuba, Puerto Rico, and elsewhere. But it was a boon for the candy business. Several factors lay the groundwork for the rise of Big Retail Candy that decade. As the 1920 Census registered, it was the first time a majority of the U.S. population was living in cities. With urbanization, higher wages and low unemployment, households began to buy more processed food. An early casualty: homemade sweets. Doughnuts, taffy and apples draped in caramel sauce began to be jettisoned in favor of store-bought candy. And then there was Prohibition: Basically, folks were swapping one vice - alcohol - for another. People at the time often said that candy and soda consumption was increasing because of Prohibition. _____, many U.S. soldiers had returned from World War I with a sweet tooth, thanks to candy included in their wartime rations. This helped spread the candy habit beyond the women and children who had previously been assumed to be the main consumers.

16. Choose the words that best fill in the blank.

- ① For this reason ② What's more ③ By and large ④ Even so

[17-18]

The Food and Drug Administration has stepped into a simmering debate in California as to whether coffee should come with a cancer warning label. In March, a judge sided with a nonprofit organization that argued that coffee contains high levels of acrylamide, a cancer-causing chemical compound produced as beans roast. Coffee companies didn't deny acrylamide's presence but argued that it was found at low levels that posed no significant health risk and was outweighed by other health benefits. That argument wasn't compelling to Los Angeles County Superior Court Judge Elihu Berle. He ordered coffee companies in California to carry a cancer warning label under Proposition 65, which requires the state to maintain a list of harmful substances and businesses to notify customers of exposure, has led to both a reduction in carcinogenic chemicals and quick settlements over labels on foods. On Wednesday, FDA Commissioner Scott Gottlieb said in a statement that if a state law purports to require food labeling to include a false or misleading statement, the FDA may decide to step in. He added that a large body of research has found little evidence that coffee causes cancer and instead suggested that it might reduce the risk of some cancers: Strong and consistent evidence shows that in healthy adults, moderate coffee consumption is not associated with an increased risk of major chronic diseases, such as cancer, or premature death, and some evidence suggests that coffee consumption may decrease the risk of certain cancers. _____, the cancer label warning, he said, may mislead consumers to believe that drinking coffee could be dangerous to their health when it actually could provide health benefits. The agency also announced that it sent a letter of support to the California Office of Environmental Health Hazard Assessment, which proposed a regulation to exempt coffee companies like Starbucks from

putting the warning label on their products.

17. Choose the words that best fill in the blank.

- ① Accordingly ② By the way ③ For instance ④ Nevertheless

18. What can be inferred from the passage above?

- ① The FDA concurs with the social enterprise on the needlessness of cancer warnings for coffee.
② The FDA fears the warning may potentially impact public perception of coffee consumption.
③ The FDA supports Proposition 65 as it reflects a scientific consensus on coffee's benefits.
④ The FDA highlights that coffee consumption lowers the risk of untimely passing.

[19-20]

A young bar-tailed godwit appears to have set a non-stop distance record for migratory birds by flying at least 13,560 kilometers from Alaska to the Australian state of Tasmania, a bird expert said Friday. The bird started on a southwestern course toward Japan then turned southeast over Alaska's Aleutian Islands. The bird was again tracking southwest when it flew over or near Kiribati and New Caledonia, then past the Australian mainland before turning directly west for Tasmania, Australia's most southerly state. The satellite trail showed it covered 13,560 kilometers without stopping. "Whether this is an accident, whether this bird got lost or whether this is part of a normal pattern of migration for the species, we still don't know," said Woehler, who is part of the research project. Guinness World Records lists the longest recorded migration by a bird without stopping for food or rest as 12,200 kilometers by a satellite-tagged male bar-tailed godwit flying from Alaska to New Zealand. That flight was recorded in 2020 as part of the same decade-old research project. The same bird broke its own record with a 13,000-kilometer flight on its next migration last year, researchers say. But Guinness has yet to acknowledge that feat. Woehler said researchers did not know whether the latest bird, known by its satellite tag 234684, flew alone or as part of a flock. Woehler said there are so few birds that have been tagged, we don't know _____. It may be that half the birds that do the migration from Alaska come to Tasmania directly rather than through New Zealand or it might be 1%, or it might be that this is the first it's ever happened. "Adult birds depart Alaska earlier than juveniles, so the tagged bird was unlikely to have followed more experienced travelers south," Woehler said. Woehler hopes to see the bird once wet weather clears in the remote corner of Tasmania, where it will fatten up having lost half its body weight on its journey.

19. Choose the best for the blank.

- ① how representative or otherwise this event is
- ② why most birds adhere to the migration patterns through New Zealand
- ③ whether that achievement should be acknowledged by Guinness
- ④ whether the bird species communicates its flight paths

20. What can be inferred from the passage above?

- ① Migrating from Alaska to New Zealand, with Australia as a waypoint, is the flying pattern of the bird species.
- ② Flying the southwest route was an incident for the species, backed by evidence.
- ③ The bird's record and the previously recorded distance differ by 1,360 kilometers.
- ④ The bird's nonstop migration from Alaska to Tasmania might be an outlier among the species.

수학

1. 함수 $y(x)$ 가 미분방정식 $x^2y'' - 2xy' + 2y = 0$ 에 대하여 조건 $y(1) = 1$ 과 $y'(1) = 3$ 을 만족할 때, $y(2)$ 의 값은? [3.8점]

① 1

② 3

③ 4

④ 6

2. 함수 $y(x)$ 가 미분방정식 $y' = 2xy^2$ 에 대하여 조건 $y(1) = \frac{1}{2}$ 을 만족할 때, $y(3)$ 의 값은? [3.8점]

① $-\frac{1}{9}$ ② $-\frac{1}{6}$ ③ $-\frac{1}{4}$ ④ $-\frac{1}{2}$

3. 다음 중 대각화가 불가능한 행렬은? [4.1점]

① $\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$

② $\begin{bmatrix} 2 & 2 \\ 0 & -3 \end{bmatrix}$

③ $\begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$

④ $\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$

4. 행렬 $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 5 \\ 3 & 4 & 5 & 2 \\ 4 & 5 & 6 & 2 \end{bmatrix}$ 의 행렬식 값은? [4.2점]

① -24

② -18

③ -12

④ -6

5. 영역 $D = \{(x, y) \mid 4x^2 + y^2 \leq 16\}$ 에서의 이변수 함수 $f(x, y) = 8x^2 + 4y$ 의 최댓값을 M , 최솟값을 m 이라 할 때, $\frac{m}{M}$ 의 값은? [4.3점]

① $-\frac{11}{34}$

② $-\frac{8}{17}$

③ $-\frac{21}{34}$

④ $-\frac{13}{17}$

6. <보기>의 급수 중 수렴하는 급수의 개수는? [4.4점]

<보기>

$\textcircled{\text{㉠}} \sum_{n=2}^{\infty} \left(\frac{n}{\ln n} \right)^n$

$\textcircled{\text{㉡}} \sum_{n=1}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdot \dots \cdot (3n-1)}{3 \cdot 7 \cdot 11 \cdot \dots \cdot (4n-1)}$

$\textcircled{\text{㉢}} \sum_{n=2}^{\infty} \frac{1}{n \ln n}$

① 0

② 1

③ 2

④ 3

7. 극한 $\lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{2x^2 \sin x}$ 의 값은? [4.5점]

① -6

② $-\frac{1}{6}$

③ $-\frac{1}{4}$

④ -4

8. 미분방정식 $(3x^2y + 2y^3)dx + (x^3 + 6xy^2)dy = 0$ 의 일반해를 $ax^my^n + bx^vy^w = C$ 라 할 때,
 $\frac{a}{b}(m+n+v+w)$ 의 값으로 가능한 것은? [4.6점]

① 1

② 2

③ 4

④ 8

9. 함수 $y(t)$ 가 미분방정식 $\frac{dy}{dt} + y - 6 \int_0^t y(\tau) d\tau = 5$ 에 대하여 조건 $y(0) = 5$ 를 만족할 때, $y(\ln 2)$ 의 값은? [4.7점]

- ① $\frac{49}{4}$ ② $\frac{65}{8}$ ③ $\frac{33}{8}$ ④ $\frac{17}{4}$

10. 영행렬이 아닌 $n \times n$ 행렬 A 와 B 에 대하여 <보기> 중 참인 명제는 모두 몇 개인가? [4.8점]

- <보기>
- ㉠ A 가 대칭행렬이고 B 가 대각행렬이면 $AB = BA$ 이다.
- ㉡ AA^T 는 대칭행렬이다.
- ㉢ $\text{tr}(A) \geq \text{tr}(B)$ 이면 $\text{tr}(A^2) \geq \text{tr}(B^2)$ 이다.
- ㉣ $\det(A) \geq \det(B)$ 이면 $\det(A^2) \geq \det(B^2)$ 이다.

- ① 1 ② 2 ③ 3 ④ 4

11. 미분방정식 $y'' - y' - 6y = xe^x$ 의 해를 $y(x) = c_1e^{ax} + c_2e^{bx} + Axe^x + Be^x$ 이라 할 때, $\frac{abA}{B}$ 의 값은?

[4.9점]

① -6

② -12

③ -36

④ -48

12. 정적분 $\int_0^1 \frac{x}{\sqrt{3-2x-x^2}} dx$ 의 값은? [5.1점]

① $\sqrt{3} - \frac{\pi}{3}$

② $\sqrt{3} - \frac{\pi}{4}$

③ $\sqrt{3} + \frac{\pi}{4}$

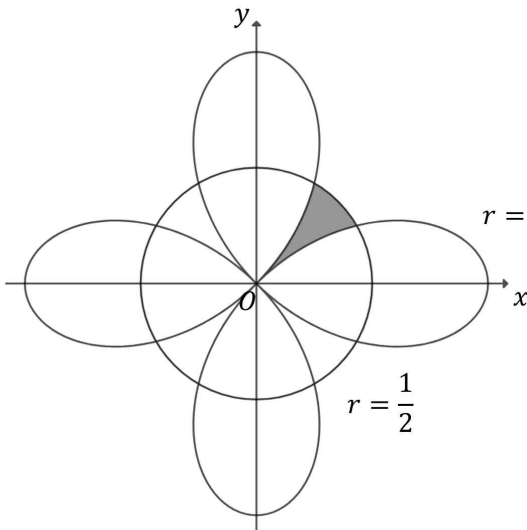
④ $\sqrt{3} + \frac{\pi}{3}$

13. 함수 $g(x) = \int_0^{\cos x} \sqrt{1 + \sin t + \cos t} dt$ 에 대하여 함수 $f(x) = \int_1^{g(x)} \frac{1}{\sqrt{1+t^3}} dt$ 일 때, $f'\left(\frac{\pi}{2}\right)$ 의 값은?

[5.3점]

- ① $-\frac{\pi}{2}$ ② $-\sqrt{2}$ ③ $-\frac{\pi}{3}$ ④ $-\frac{\sqrt{2}}{2}$

14. 극좌표로 정의된 두 곡선 $r = \cos 2\theta$ 와 $r = \frac{1}{2}$ 로 만들어지는 영역 중 색칠된 영역의 넓이는? [5.4점]



- ① $\frac{3\sqrt{3}+\pi}{48}$ ② $\frac{-4+\sqrt{3}+\pi}{16}$
 ③ $\frac{-8+3\sqrt{3}+\pi}{16}$ ④ $\frac{3\sqrt{3}-\pi}{48}$

15. 벡터장 $\mathbf{F}(x, y, z) = z \sin^{-1}(y^3)\mathbf{i} + z^3 \ln(x^{100} + 1)\mathbf{j} + z^2 \mathbf{k}$ 에 대하여, 포물면 $x^2 + y^2 + z = 3$ 과 평면 $z = 2$ 로 둘러싸인 영역에서의 선다발(flux)을 구하면? (단, 영역의 방향은 바깥쪽 방향이다.) [5.5점]

- ① $\frac{7}{3}\pi$ ② $\frac{13}{3}\pi$ ③ $\frac{19}{3}\pi$ ④ $\frac{25}{3}\pi$

16. 벡터장 $\mathbf{F}(x, y) = \left(\frac{x^5 + x^3 y^2 - y}{x^2 + y^2} \right) \mathbf{i} + \left(\frac{x^2 \cos y + y^2 \cos y + x}{x^2 + y^2} \right) \mathbf{j}$ 에 대하여 선적분 $\int_C \mathbf{F} \cdot d\mathbf{r}$ 을 구하면?

(단, C 는 타원 $\frac{x^2}{4} + \frac{y^2}{9} = 1$ 이며 양의 방향의 곡선이다.) [5.7점]

- ① π ② 2π ③ 3π ④ 4π

17. 행렬 $A = \begin{bmatrix} 2 & 5 \\ -1 & 8 \end{bmatrix}$, 벡터 $\mathbf{b} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ 와 모든 성분이 실수인 벡터 $\mathbf{x}$ 에 대하여 $\mathbf{x}^T A \mathbf{x} + \mathbf{x}^T \mathbf{b}$ 가 최소가 되게 하는 벡터 $\mathbf{x}$ 의 모든 성분의 합은? [5.9점]

- ① $\frac{3}{2}$ ② $-\frac{5}{2}$ ③ $-\frac{3}{2}$ ④ $\frac{1}{2}$

18. 고윳값으로 2와 3을 갖는 3×3 행렬 A 에 대하여, 고윳값 2에 대한 고유벡터는 $\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ 이고

고윳값 3에 대한 고유벡터는 $\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$ 일 때, $A^3 \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}$ 의 모든 성분의 합은? [6.2점]

- ① 16 ② 27 ③ 35 ④ 42

19. 반복적분 $\int_0^1 \int_{\sin^{-1}y}^{\frac{\pi}{2}} (\cos x) \sqrt{3 + \cos^2 x} \, dx dy$ 의 값은? [6.3점]

① $\frac{4}{3} - \sqrt{2}$

② $\frac{8}{3} - \sqrt{2}$

③ $\frac{4}{3} - \sqrt{3}$

④ $\frac{8}{3} - \sqrt{3}$

20. 점 $P(0, 0, 1) \in \mathbb{R}^3$ 에서 삼변수 함수 $f(x, y, z) = 3z + e^{x^2 - y^2}$ 의 최대변화율을 M 이라 하자. 점 P 에서의 f 의 방향도함수의 값이 $\frac{M}{3}$ 이 되는 모든 벡터 $\mathbf{v}$ 에 대하여, $\frac{\mathbf{v}}{|\mathbf{v}|}$ 의 끝점이 만드는 도형의 둘레의 길이는? [6.5점]

① π

② $\frac{2\sqrt{2}}{3}\pi$

③ $\frac{4\sqrt{2}}{3}\pi$

④ $\frac{8}{9}\pi$